# Chapter 13: Oscillations - NCERT Exercise Solutions

## Detailed Answer Key for All Back Exercise Questions

### Class 11 Physics - Complete Solutions with Explanations

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## 13.1 Which of the following examples represent periodic motion?

### (a) A swimmer completing one (return) trip from one bank of a river to the other and back.

**Answer: NO - Not periodic motion**

**Explanation:**

- The swimmer completes one cycle and then stops
- For periodic motion, the motion must repeat continuously at regular intervals
- A single round trip, even if it could be repeated, doesn't constitute periodic motion by itself
- Periodic motion requires infinite repetition of the same pattern

### (b) A freely suspended bar magnet displaced from its N-S direction and released.

**Answer: YES - Periodic motion**

**Explanation:**

- When displaced from its equilibrium position (N-S direction), the bar magnet experiences a restoring torque
- Earth's magnetic field provides the restoring force
- The magnet oscillates about its equilibrium position

- This oscillation repeats at regular intervals
- **Period:** Depends on the magnet's moment of inertia and Earth's magnetic field strength

### (c) A hydrogen molecule rotating about its centre of mass.

**Answer: YES - Periodic motion**

**Explanation:**

- Molecular rotation is repetitive motion
- The molecule completes one full rotation and returns to its initial configuration
- This rotation continues indefinitely under normal conditions
- **Period:** Time for one complete rotation (very small, typically $10^{-12}$ seconds)

### (d) An arrow released from a bow.

**Answer: NO - Not periodic motion**

**Explanation:**

- The arrow follows a parabolic trajectory (projectile motion)
- Once released, it doesn't return to repeat the same path
- It's a one-time motion from bow to target
- No repetitive pattern exists

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## 13.2 Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?

### (a) The rotation of earth about its axis.

**Answer: Periodic but NOT simple harmonic motion**

**Explanation:**

- **Periodic:** Earth completes one rotation every 24 hours
- **Not SHM:**
  - No restoring force proportional to displacement
  - No equilibrium position about which it oscillates
  - Circular motion, not oscillatory motion
  - Angular displacement varies linearly with time, not sinusoidally

### (b) Motion of an oscillating mercury column in a U-tube.

**Answer: Nearly simple harmonic motion**

**Explanation:**

- **Equilibrium position:** When mercury levels are equal in both arms
- **Restoring force:** Pressure difference due to height difference
- **Force analysis:** $F = -\rho gA(2x) = -2\rho gAx$
- **Force law:** $F \propto -x$ (proportional to displacement)
- **SHM equation:** $a = -(2g/L)x$, where L is total mercury column length
- **Period:** $T = 2\pi\sqrt{(L/2g)}$
- **"Nearly" because:** Small friction and viscous effects cause slight deviations

### (c) Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lowermost point.

**Answer: Nearly simple harmonic motion**

**Explanation:**

- **Equilibrium position:** Bottom of the bowl
- **For small displacements:** Restoring force $F = -mg\sin\theta \approx -mg\theta \approx -mg(x/R)$
- **Force law:** $F = -(mg/R)x$, which is $F \propto -x$
- **SHM condition satisfied:** Force proportional to displacement
- **Period:** $T = 2\pi\sqrt{(R/g)}$, where R is radius of curvature at bottom
- **"Nearly" because:**
  - Valid only for small oscillations
  - Assumes perfectly smooth bowl
  - Real bowls have slight friction

### (d) General vibrations of a polyatomic molecule about its equilibrium position.

**Answer: Periodic but NOT simple harmonic motion**

**Explanation:**

- **Periodic:** Molecular vibrations are repetitive
- **Not simple harmonic:**
  - Multiple atoms with complex interactions
  - Multiple normal modes of vibration
  - Superposition of many different frequencies
  - Anharmonic terms in potential energy
  - Non-linear restoring forces at larger amplitudes
- **Result:** Complex periodic motion that cannot be described by single SHM equation

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## 13.3 Fig. 13.18 depicts four x-t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?

### Analysis of Each Plot:

**Plot (a):**

- **Answer: YES - Periodic motion**
- **Period: T = 2 seconds**
- **Explanation:** The displacement pattern repeats exactly every 2 seconds. At t = 0, 2, 4, 6... the displacement is the same, and the entire waveform repeats.

**Plot (b):**

- **Answer: NO - Not periodic motion**
- **Explanation:** This appears to be exponential decay or growth. The displacement continuously increases/decreases without repetition. No repeating pattern exists.

**Plot (c):**

- **Answer: YES - Periodic motion**
- **Period: T = 4 seconds**
- **Explanation:** The triangular wave pattern repeats every 4 seconds. The displacement varies linearly in segments, but the overall pattern is periodic.

**Plot (d):**

- **Answer: NO - Not periodic motion**
- **Explanation:** This shows a step function with random or non-repeating steps. There's no regular interval after which the motion repeats itself.

### Key Points for Identifying Periodic Motion:

1. **Mathematical test:** $f(t) = f(t + T)$ for all t
2. **Visual test:** The graph should look identical when shifted by period T
3. **Smallest period:** Find the smallest T for which repetition occurs

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## 13.4 Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):

### (a) sin ωt – cos ωt

**Answer: Simple harmonic motion**

**Detailed Solution:**

$$\sin \omega t - \cos \omega t = \sin \omega t - \sin(\pi/2 - \omega t)$$
$$= 2 \cos(\pi/4) \sin(\omega t - \pi/4)$$
$$= \sqrt{2} \sin(\omega t - \pi/4)$$

**Alternative form:**

$$\sin \omega t - \cos \omega t = \sqrt{2} \sin(\omega t - \pi/4) = \sqrt{2} \cos(\omega t - \pi/4 - \pi/2)$$
$$= \sqrt{2} \cos(\omega t - 3\pi/4)$$

- **Form:** $A \cos(\omega t + \varphi)$ where $A = \sqrt{2}$, $\varphi = -3\pi/4$
- **Period:** $T = 2\pi/\omega$
- **This is standard SHM**

### (b) $\sin^3 \omega t$

**Answer: Periodic but not simple harmonic**

**Detailed Solution:** Using trigonometric identity:

$$\sin^3 \omega t = (3 \sin \omega t - \sin 3\omega t)/4$$

**Analysis:**

- Contains $\sin \omega t$ (period $2\pi/\omega$) and $\sin 3\omega t$ (period $2\pi/3\omega$)
- LCM of periods = $2\pi/\omega$
- **Period: $T = 2\pi/\omega$**
- **Not SHM:** Contains harmonics ($\sin 3\omega t$ term)
- **Periodic:** Repeats every $2\pi/\omega$

### (c) $3 \cos (\pi/4 – 2\omega t)$

**Answer: Simple harmonic motion**

**Detailed Solution:**

$$3 \cos(\pi/4 - 2\omega t) = 3 \cos(2\omega t - \pi/4)$$

- **Standard SHM form:** $A \cos(\omega t + \varphi)$
- **Amplitude:** $A = 3$
- **Angular frequency:** $2\omega$
- **Phase constant:** $\varphi = -\pi/4$
- **Period:** $T = 2\pi/(2\omega) = \pi/\omega$

### (d) $\cos \omega t + \cos 3\omega t + \cos 5\omega t$

**Answer: Periodic but not simple harmonic**

**Detailed Solution:**

- **Individual periods:**
  - $\cos \omega t$: $T_1 = 2\pi/\omega$
  - $\cos 3\omega t$: $T_2 = 2\pi/3\omega$
  - $\cos 5\omega t$: $T_3 = 2\pi/5\omega$
- **Finding common period:**
  - $T_1 = 2\pi/\omega$
  - $T_2 = 2\pi/3\omega = T_1/3$
  - $T_3 = 2\pi/5\omega = T_1/5$
- **LCM calculation:** Since 3 and 5 are prime, LCM = $T_1$
- **Period: $T = 2\pi/\omega$**
- **Not SHM:** Superposition of multiple harmonics
- **Periodic:** All component frequencies are integer multiples of fundamental

### (e) $\exp(-\omega^2 t^2)$

**Answer: Non-periodic motion**

**Detailed Solution:**

- **Function:** $e^{(-\omega^2 t^2)}$ (Gaussian function)
- **Behavior:** Monotonically decreases from 1 at $t = 0$ to 0 as $t \to \infty$
- **Test for periodicity:** $f(t + T) = f(t)$?
  - $e^{(-\omega^2(t+T)^2)} = e^{(-\omega^2 t^2)}$?

- This is impossible for any finite $T > 0$

- **Physical interpretation:** Represents exponential damping, not oscillation

### (f) $1 + \omega t + \omega^2 t^2$

**Answer: Non-periodic motion**

**Detailed Solution:**

- **Function type:** Quadratic polynomial in time
- **Behavior:** Continuously increasing (for $\omega > 0$)
- **Test for periodicity:** $f(t + T) = f(t)$?
  - $1 + \omega(t + T) + \omega^2(t + T)^2 = 1 + \omega t + \omega^2 t^2$?
  - This gives: $\omega T + \omega^2(2tT + T^2) = 0$
  - For this to hold for all t, we need $\omega = 0$, which contradicts given condition
- **Physical interpretation:** Uniformly accelerated motion starting from rest

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## 13.5 A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is:

### Setup:

- **Distance AB = 10 cm**
- **Mean position O:** At center, 5 cm from each end
- **Amplitude A = 5 cm**
- **Coordinate system:** A at $x = -5$ cm, B at $x = +5$ cm, O at $x = 0$

- **SHM equation:** $x = A \cos(\omega t + \varphi)$
- **Force law:** $F = -kx$ (always toward mean position)

### (a) At the end A

**Position:** $x = -5$ cm (extreme left position)

**Velocity:**

- At extreme positions, velocity = 0
- **Sign: v = 0** (neither positive nor negative)

**Acceleration:**

- $a = -\omega^2 x = -\omega^2(-5) = +5\omega^2$ cm/s²
- **Sign: Positive** (toward mean position, rightward)

**Force:**

- $F = -kx = -k(-5) = +5k$
- **Sign: Positive** (rightward, toward equilibrium)

### (b) At the end B

**Position:** $x = +5$ cm (extreme right position)

**Velocity:**

- At extreme positions, velocity = 0
- **Sign: v = 0** (neither positive nor negative)

**Acceleration:**

- $a = -\omega^2 x = -\omega^2(+5) = -5\omega^2$ cm/s²

- **Sign: Negative** (toward mean position, leftward)

**Force:**

- $F = -kx = -k(+5) = -5k$
- **Sign: Negative** (leftward, toward equilibrium)

### (c) At the mid-point of AB going towards A

**Position:** $x = 0$ (mean position)

**Velocity:**

- Maximum speed at mean position
- Moving toward A means moving in negative direction
- **Sign: Negative** (leftward)

**Acceleration:**

- $a = -\omega^2 x = -\omega^2(0) = 0$
- **Sign: Zero** (no acceleration at mean position)

**Force:**

- $F = -kx = -k(0) = 0$
- **Sign: Zero** (no force at equilibrium position)

### (d) At 2 cm away from B going towards A

**Position:** $x = +3$ cm (2 cm from B toward center)

**Velocity:**

- Moving toward A means negative velocity

- **Sign: Negative** (leftward motion)

**Acceleration:**

- $a = -\omega^2 x = -\omega^2(+3) = -3\omega^2$ cm/s$^2$
- **Sign: Negative** (toward mean position, leftward)

**Force:**

- $F = -kx = -k(+3) = -3k$
- **Sign: Negative** (toward equilibrium, leftward)

### (e) At 3 cm away from A going towards B

**Position:** $x = -2$ cm (3 cm from A toward center)

**Velocity:**

- Moving toward B means positive velocity
- **Sign: Positive** (rightward motion)

**Acceleration:**

- $a = -\omega^2 x = -\omega^2(-2) = +2\omega^2$ cm/s$^2$
- **Sign: Positive** (toward mean position, rightward)

**Force:**

- $F = -kx = -k(-2) = +2k$
- **Sign: Positive** (toward equilibrium, rightward)

### (f) At 4 cm away from B going towards A

**Position:** $x = +1$ cm (4 cm from B toward center)

**Velocity:**

- Moving toward A means negative velocity
- **Sign: Negative** (leftward motion)

**Acceleration:**

- $a = -\omega^2 x = -\omega^2(+1) = -\omega^2$ cm/s$^2$
- **Sign: Negative** (toward mean position, leftward)

**Force:**

- $F = -kx = -k(+1) = -k$
- **Sign: Negative** (toward equilibrium, leftward)

## Summary Table:

| Position | Velocity | Acceleration | Force |
|---|---|---|---|
| (a) End A | 0 | Positive | Positive |
| (b) End B | 0 | Negative | Negative |
| (c) Middle→A | Negative | 0 | 0 |
| (d) 2cm from B→A | Negative | Negative | Negative |
| (e) 3cm from A→B | Positive | Positive | Positive |
| (f) 4cm from B→A | Negative | Negative | Negative |

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## 13.6 Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?

### Condition for SHM:

**$a = -\omega^2 x$** (acceleration proportional to displacement, opposite in direction)

### (a) $a = 0.7x$

**Answer: NOT simple harmonic motion**

**Explanation:**

- Acceleration is proportional to displacement
- **Problem:** Same direction as displacement (positive proportionality)
- This leads to **exponential growth**, not oscillatory motion
- **Mathematical solution:** $x = Ae^{(\sqrt{0.7} \cdot t)} + Be^{(-\sqrt{0.7} \cdot t)}$
- **Physical result:** Unstable motion, particle moves away from equilibrium

### (b) $a = -200x^2$

**Answer: NOT simple harmonic motion**

**Explanation:**

- Acceleration proportional to **square** of displacement
- **Non-linear relationship:** $a \propto x^2$ instead of $a \propto x$
- **Result:** Non-harmonic oscillation (anharmonic motion)
- **Period:** Would depend on amplitude (unlike SHM)
- **Motion type:** Non-linear oscillator

### (c) $a = -10x$

**Answer: YES - Simple harmonic motion**

**Explanation:**

- **Perfect SHM form:** $a = -\omega^2 x$
- **Comparing:** $\omega^2 = 10$, so $\omega = \sqrt{10}$ rad/s
- **Period:** $T = 2\pi/\omega = 2\pi/\sqrt{10}$ s
- **Frequency:** $f = \sqrt{10}/2\pi$ Hz
- **General solution:** $x(t) = A\cos(\sqrt{10}\cdot t + \varphi)$

### (d) $a = 100x^3$

**Answer: NOT simple harmonic motion**

**Explanation:**

- Acceleration proportional to **cube** of displacement
- **Wrong direction:** Acceleration in same direction as displacement
- **Non-linear:** Highly non-linear relationship
- **Result:** Unstable, non-oscillatory motion
- **Mathematical behavior:** Exponential divergence

## Key Points:

1. **SHM requirement:** $a = -\omega^2 x$ (linear, negative proportionality)
2. **Sign importance:** Negative sign ensures restoring nature
3. **Linearity:** Must be first power of x
4. **Only option (c) satisfies all SHM conditions**

## 13.7 The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t) = A \cos(\omega t + \varphi)$. If the initial ($t = 0$) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is $\pi\ s^{-1}$. If instead of the cosine function, we choose the sine function to describe the SHM: $x = B \sin(\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions.

### Given Data:

- $x(0) = 1$ cm
- $v(0) = \omega$ cm/s $= \pi$ cm/s (since $\omega = \pi\ s^{-1}$)
- $\omega = \pi\ s^{-1}$

### Part A: Cosine Function $x(t) = A \cos(\omega t + \varphi)$

#### Step 1: Apply initial position condition

$x(0) = A \cos(\varphi) = 1$ cm ... (1)

#### Step 2: Find velocity function

$v(t) = dx/dt = -A\omega \sin(\omega t + \varphi)$

#### Step 3: Apply initial velocity condition

$$v(0) = -A\omega \sin(\varphi) = \pi \text{ cm/s}$$
$$-A\pi \sin(\varphi) = \pi$$
$$-A \sin(\varphi) = 1$$
$$A \sin(\varphi) = -1 \quad \ldots (2)$$

**Step 4: Solve for A and φ**

From equations (1) and (2):

$$A \cos(\varphi) = 1$$
$$A \sin(\varphi) = -1$$

**Finding amplitude A:**

$$A^2 = (A \cos \varphi)^2 + (A \sin \varphi)^2$$
$$A^2 = 1^2 + (-1)^2$$
$$A^2 = 2$$
$$A = \sqrt{2} \text{ cm}$$

**Finding phase angle φ:**

$$\tan(\varphi) = \sin(\varphi)/\cos(\varphi) = -1/1 = -1$$
$$\varphi = -\pi/4 \text{ or } 3\pi/4$$

**Determining correct φ:** Since $A \cos(\varphi) = 1 > 0$ and $A \sin(\varphi) = -1 < 0$, we need φ in fourth quadrant.

$$\varphi = -\pi/4 \text{ rad}$$

**Verification:**

- $A \cos(-\pi/4) = \sqrt{2} \times (1/\sqrt{2}) = 1$ ✓
- $A \sin(-\pi/4) = \sqrt{2} \times (-1/\sqrt{2}) = -1$ ✓

## Part B: Sine Function $x = B \sin(\omega t + \alpha)$

### Step 1: Apply initial position condition

$$x(0) = B \sin(\alpha) = 1 \text{ cm} \quad \ldots (3)$$

### Step 2: Find velocity function

$$v(t) = dx/dt = B\omega \cos(\omega t + \alpha)$$

### Step 3: Apply initial velocity condition

$$v(0) = B\omega \cos(\alpha) = \pi \text{ cm/s}$$
$$B\pi \cos(\alpha) = \pi$$
$$B \cos(\alpha) = 1 \quad \ldots (4)$$

### Step 4: Solve for B and α

From equations (3) and (4):

$$B \sin(\alpha) = 1$$
$$B \cos(\alpha) = 1$$

**Finding amplitude B:**

$$B^2 = (B \sin \alpha)^2 + (B \cos \alpha)^2$$
$$B^2 = 1^2 + 1^2$$
$$B^2 = 2$$
$$B = \sqrt{2} \text{ cm}$$

**Finding phase angle α:**

$$\tan(\alpha) = \sin(\alpha)/\cos(\alpha) = 1/1 = 1$$
$\alpha = \pi/4$ rad (first quadrant since both $\sin \alpha > 0$ and $\cos \alpha > 0$)

**Verification:**

- $B \sin(\pi/4) = \sqrt{2} \times (1/\sqrt{2}) = 1$ ✓
- $B \cos(\pi/4) = \sqrt{2} \times (1/\sqrt{2}) = 1$ ✓

### Final Answers:

- **Cosine form:** $A = \sqrt{2}$ cm, $\varphi = -\pi/4$ rad
- **Sine form:** $B = \sqrt{2}$ cm, $\alpha = \pi/4$ rad

### Relationship between the two forms:

$$x = \sqrt{2} \cos(\pi t - \pi/4) = \sqrt{2} \sin(\pi t + \pi/4)$$

This shows that $\cos(\theta - \pi/4) = \sin(\theta + \pi/4)$, which is a standard trigonometric identity.

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## 13.8 A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced

**and released, oscillates with a period of 0.6 s. What is the weight of the body?**

### Given Data:

- Maximum reading = 50 kg
- Scale length = 20 cm = 0.2 m
- Period of oscillation T = 0.6 s
- g = 9.8 m/s² (standard gravity)

### Understanding the Problem:

**Spring balance working principle:**

- Extension is proportional to applied force (Hooke's law)
- Maximum extension (0.2 m) corresponds to maximum weight (50 kg)

**Finding spring constant:**

At maximum load:

```
F_max = k × x_max
50 kg × g = k × 0.2 m
k = (50 × 9.8)/0.2 = 2450 N/m
```

### Solution Method:

**For oscillating spring-mass system:**

```
T = 2π√(m/k)
```

**Solving for mass:**

$$T^2 = 4\pi^2 m/k$$
$$m = kT^2/(4\pi^2)$$

**Substituting values:**

$$m = 2450 \times (0.6)^2/(4\pi^2)$$
$$m = 2450 \times 0.36/(4 \times 9.87)$$
$$m = 882/(39.48)$$
$$m = 22.35 \text{ kg}$$

**Finding weight:**

$$W = mg = 22.35 \times 9.8 = 219 \text{ N}$$

**Converting to kg-force:**

Weight = 22.35 kg (in terms of mass)

## Verification:

**Check using period formula:**

$$T = 2\pi\sqrt{(22.35/2450)} = 2\pi\sqrt{(0.00912)} = 2\pi \times 0.0955 = 0.6 \text{ s} ✓$$

**Physical reasonableness:**

- The body weighs 22.35 kg, which is less than the maximum capacity (50 kg) ✓
- The oscillation period is reasonable for this mass-spring system ✓

**Answer: The weight of the body is 22.35 kg (or 219 N)**

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## 13.9 A spring having spring constant 1200 N $m^{-1}$ is mounted on a horizontal table as shown in Fig. 13.19. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released.

### Given Data:

- Spring constant k = 1200 N/m
- Mass m = 3 kg
- Initial displacement A = 2.0 cm = 0.02 m
- Horizontal system (no gravity effects on oscillation)

### (i) Find the frequency of oscillations

**Angular frequency:**

$$\omega = \sqrt{(k/m)} = \sqrt{(1200/3)} = \sqrt{400} = 20 \text{ rad/s}$$

**Frequency:**

$$f = \omega/(2\pi) = 20/(2\pi) = 10/\pi = 3.18 \text{ Hz}$$

**Alternative calculation:**

$$\text{Period } T = 2\pi\sqrt{(m/k)} = 2\pi\sqrt{(3/1200)} = 2\pi\sqrt{(0.0025)} = 2\pi \times 0.05 = 0.314 \text{ s}$$
$$\text{Frequency } f = 1/T = 1/0.314 = 3.18 \text{ Hz}$$

## (ii) Find maximum acceleration of the mass

**Maximum acceleration occurs at extreme positions:**

$$a_{max} = \omega^2 A = (20)^2 \times 0.02 = 400 \times 0.02 = 8.0 \text{ m/s}^2$$

**Alternative method using force:**

$$F_{max} = kA = 1200 \times 0.02 = 24 \text{ N}$$
$$a_{max} = F_{max}/m = 24/3 = 8.0 \text{ m/s}^2$$

**Direction:** Always toward the mean position

## (iii) Find the maximum speed of the mass

**Maximum speed occurs at mean position:**

$$v_{max} = \omega A = 20 \times 0.02 = 0.4 \text{ m/s}$$

**Alternative method using energy:**

$$\text{Total energy } E = \tfrac{1}{2}kA^2 = \tfrac{1}{2} \times 1200 \times (0.02)^2 = 0.24 \text{ J}$$
$$\text{At mean position: } E = \tfrac{1}{2}mv_{max}^2$$
$$0.24 = \tfrac{1}{2} \times 3 \times v_{max}^2$$
$$v_{max}^2 = 0.48/3 = 0.16$$
$$v_{max} = 0.4 \text{ m/s}$$

## Summary of Results:

- **(i) Frequency:** $f = 3.18$ Hz

- **(ii) Maximum acceleration:** $a_{max} = 8.0\ m/s^2$
- **(iii) Maximum speed:** $v_{max} = 0.4\ m/s$

### Physical Interpretation:

- **High frequency** due to stiff spring and moderate mass
- **Maximum acceleration** occurs when spring force is maximum
- **Maximum speed** occurs when all potential energy converts to kinetic energy

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## 13.10 In Exercise 13.9, let us take the position of mass when the spring is unstretched as x = 0, and the direction from left to right as the positive direction of x-axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch (t = 0), the mass is:

### Given from Previous Problem:

- $k = 1200$ N/m, $m = 3$ kg, $A = 0.02$ m
- $\omega = 20$ rad/s
- Coordinate system: $x = 0$ at natural length, rightward positive

### (a) At the mean position

**Initial conditions at t = 0:**

- Position: $x(0) = 0$ (at equilibrium)
- Released from rest, so $v(0) = 0$

**General SHM equation:**

$$x(t) = A\cos(\omega t + \varphi)$$
$$v(t) = -A\omega\sin(\omega t + \varphi)$$

**Applying initial conditions:**

$$x(0) = A\cos(\varphi) = 0$$
This gives: $\cos(\varphi) = 0$
Therefore: $\varphi = \pm\pi/2$

$$v(0) = -A\omega\sin(\varphi) = 0$$
For $\varphi = \pi/2$: $\sin(\pi/2) = 1$, so $v(0) = -A\omega \neq 0$
For $\varphi = -\pi/2$: $\sin(-\pi/2) = -1$, so $v(0) = A\omega \neq 0$

**Problem analysis:** This situation is impossible because if the mass starts at mean position with zero velocity, it will remain at rest (no force acts on it).

**Correct interpretation:** The mass must have some initial velocity to oscillate. Assuming it starts at mean position with maximum velocity:

$x(0) = 0$ and $v(0) = \pm v_{max} = \pm\omega A$

**For rightward initial velocity:**

$x(t) = A\sin(\omega t) = 0.02\sin(20t)$ meters

**For leftward initial velocity:**

$x(t) = -A\sin(\omega t) = -0.02\sin(20t)$ meters

## (b) At the maximum stretched position

**Initial conditions at t = 0:**

- Position: $x(0) = A = 0.02$ m (maximum rightward displacement)
- Released from rest: $v(0) = 0$

**Applying conditions:**

$$x(0) = A\cos(\varphi) = A$$
$$\cos(\varphi) = 1$$
$$\varphi = 0$$

**Verification with velocity:**

$$v(0) = -A\omega\sin(0) = 0 \checkmark$$

**Final equation:**

$$x(t) = A\cos(\omega t) = 0.02\cos(20t) \text{ meters}$$

## (c) At the maximum compressed position

**Initial conditions at t = 0:**

- Position: $x(0) = -A = -0.02$ m (maximum leftward displacement)
- Released from rest: $v(0) = 0$

**Applying conditions:**

$$x(0) = A\cos(\varphi) = -A$$
$$\cos(\varphi) = -1$$
$$\varphi = \pi$$

**Verification with velocity:**

$$v(0) = -A\omega \sin(\pi) = 0 \checkmark$$

**Final equation:**

$$x(t) = A\cos(\omega t + \pi) = -A\cos(\omega t) = -0.02\cos(20t) \text{ meters}$$

## Summary of Functions:

| Initial Condition | Function x(t) |
|---|---|
| (a) Mean position* | $x(t) = \pm 0.02 \sin(20t)$ m |
| (b) Maximum stretched | $x(t) = 0.02 \cos(20t)$ m |
| (c) Maximum compressed | $x(t) = -0.02 \cos(20t)$ m |

*Note: Case (a) requires initial velocity to oscillate

## How do these functions differ?

**Frequency:**

- All have same $\omega = 20$ rad/s
- All have same period $T = \pi/10$ s
- **No difference in frequency**

**Amplitude:**

- All have same amplitude A = 0.02 m
- **No difference in amplitude**

**Initial Phase:**

- Case (a): $\varphi = \pm\pi/2$ (sine function)
- Case (b): $\varphi = 0$ (cosine function)
- Case (c): $\varphi = \pi$ (negative cosine function)
- **Only initial phase differs**

**Physical meaning:** All represent the same oscillation starting at different points in the cycle. The phase difference determines where in the oscillation cycle we start our observation.

---

## 13.11 Figures 13.20 correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution are indicated on each figure. Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P in each case.

### Figure (a) Analysis:

- **Radius:** R = 3 cm
- **Period:** T = 2 s
- **Initial position:** At +3 cm on x-axis (0° from positive x-axis)
- **Sense of revolution:** Anticlockwise

**Angular frequency:**

$$\omega = 2\pi/T = 2\pi/2 = \pi \text{ rad/s}$$

**At t = 0:** Particle P is at (3, 0), so angle = 0

**General equation for anticlockwise motion:**

$$x(t) = R\cos(\omega t + \varphi_0)$$

where $\varphi_0$ is initial phase angle.

**Initial condition:**

$$x(0) = 3\cos(\varphi_0) = 3 \text{ cm}$$
$$\cos(\varphi_0) = 1$$
$$\varphi_0 = 0$$

**Final SHM equation:**

$$x(t) = 3\cos(\pi t) \text{ cm}$$

## Figure (b) Analysis:

- **Radius:** R = 2 cm
- **Period:** T = 4 s
- **Initial position:** At top of circle (0, +2) - 90° from positive x-axis
- **Sense of revolution:** Clockwise

**Angular frequency:**

$$\omega = 2\pi/T = 2\pi/4 = \pi/2 \text{ rad/s}$$

**For clockwise motion, angle decreases with time:**

$$\theta(t) = \theta_0 - \omega t$$

**At t = 0:** Particle P is at (0, 2), so initial angle $\theta_0 = \pi/2$

**Position equations:**

$$x(t) = R\cos(\theta_0 - \omega t) = R\cos(\pi/2 - \pi t/2)$$
$$x(t) = R\sin(\pi t/2) = 2\sin(\pi t/2) \text{ cm}$$

**Alternative form:**

$$x(t) = 2\sin(\pi t/2) = 2\cos(\pi t/2 - \pi/2) \text{ cm}$$

## Summary:

- **Figure (a):** $x(t) = 3\cos(\pi t)$ cm
- **Figure (b):** $x(t) = 2\sin(\pi t/2)$ cm

---

## 13.12 Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial ($t = 0$) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

### (a) $x = -2 \sin(3t + \pi/3)$

**Converting to standard cosine form:**

$$x = -2 \sin(3t + \pi/3)$$
$$x = 2 \sin(3t + \pi/3 + \pi) \quad [\text{since } -\sin \theta = \sin(\theta + \pi)]$$
$$x = 2 \sin(3t + 4\pi/3)$$
$$x = 2 \cos(3t + 4\pi/3 - \pi/2) \quad [\text{since } \sin \theta = \cos(\theta - \pi/2)]$$
$$x = 2 \cos(3t + 5\pi/3)$$

**Reference Circle Parameters:**

- **Radius:** $A = 2$ cm
- **Angular speed:** $\omega = 3$ rad/s
- **Initial phase:** $\varphi = 5\pi/3 = -\pi/3$ (equivalent angles)
- **Initial position at $t = 0$:** $x(0) = 2 \cos(5\pi/3) = 2 \cos(-\pi/3) = 2 \times (1/2) = 1$ cm

**Reference Circle Description:**

- Circle of radius 2 cm
- Particle rotates anticlockwise at 3 rad/s
- At $t = 0$, particle at angle $-\pi/3$ (or $5\pi/3$) from positive x-axis
- Initial x-projection = 1 cm

### (b) $x = \cos(\pi/6 - t)$

**Converting to standard form:**

$$x = \cos(\pi/6 - t) = \cos(-t + \pi/6) = \cos(-(t - \pi/6))$$
$$x = \cos(t - \pi/6) \quad [\text{since } \cos(-\theta) = \cos(\theta)]$$

**But for anticlockwise motion, we need positive ω:**

$$x = \cos(-(t - \pi/6)) = \cos(-t + \pi/6)$$

**This represents clockwise motion. For anticlockwise equivalent:**

$$x = \cos(-t + \pi/6) = \cos(t - \pi/6 + \pi) = \cos(t + 5\pi/6)$$

**Wait, let's be more careful:**

$$x = \cos(\pi/6 - t)$$

**This can be written as:**

$$x = \cos(-(t - \pi/6)) = \cos(t - \pi/6)$$

**For anticlockwise motion with negative ω coefficient, we interpret:** $\omega = -1$ means clockwise at 1 rad/s, equivalent to anticlockwise at 1 rad/s with phase shift π.

**Reference Circle Parameters:**

- **Radius:** A = 1 cm

- **Angular speed:** $\omega = 1$ rad/s
- **At t = 0:** $x(0) = \cos(\pi/6) = \sqrt{3}/2 \approx 0.866$ cm
- **Initial angle:** $\pi/6$ from positive x-axis

### (c) x = 3 sin(2πt + π/4)

**Converting to cosine form:**

```
x = 3 sin(2πt + π/4)
x = 3 cos(2πt + π/4 - π/2)
x = 3 cos(2πt - π/4)
```

**Reference Circle Parameters:**

- **Radius:** $A = 3$ cm
- **Angular speed:** $\omega = 2\pi$ rad/s
- **Initial phase:** $\varphi = -\pi/4$
- **Initial position at t = 0:** $x(0) = 3\cos(-\pi/4) = 3 \times (1/\sqrt{2}) = 3/\sqrt{2} \approx 2.12$ cm

### (d) x = 2 cos πt

**Already in standard form:**

```
x = 2 cos(πt + 0)
```

**Reference Circle Parameters:**

- **Radius:** $A = 2$ cm
- **Angular speed:** $\omega = \pi$ rad/s
- **Initial phase:** $\varphi = 0$

- **Initial position at t = 0:** $x(0) = 2\cos(0) = 2$ cm

**Summary Table:**

| Motion | Radius (cm) | Angular Speed (rad/s) | Initial Phase | Initial x-position (cm) |
|---|---|---|---|---|
| (a) | 2 | 3 | 5π/3 | 1 |
| (b) | 1 | 1 | π/6 | √3/2 ≈ 0.866 |
| (c) | 3 | 2π | -π/4 | 3/√2 ≈ 2.12 |
| (d) | 2 | π | 0 | 2 |

---

## 13.13 Figure 13.21(a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure 13.21 (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Fig. 13.21(b) is stretched by the same force F.

### (a) What is the maximum extension of the spring in the two cases?

**Case (a): One end fixed, one end with mass m**

**Force balance:**

```
Applied force F = Spring restoring force
F = kx₁
x₁ = F/k
```

**Maximum extension in case (a):** $x_1 = F/k$

**Case (b): Both ends free, masses m at each end**

**Analysis:**

- Both masses experience same force F
- Spring extends on both sides
- Each half of spring acts independently
- Consider spring as two springs of constant k each (since it's the same spring)

**For each mass:**

$$F = k'x_2$$

**However, we need to find effective spring constant:** When both ends are pulled with force F, the total extension is $x_2$ on each side.

**Key insight:** The middle of the spring doesn't move due to symmetry. This is equivalent to having two springs of length L/2 each, connected in parallel.

**Wait, this needs careful analysis:**

**Method 1 - Energy consideration:** In case (b), both masses move distance $x_2$. The work done by both forces equals the elastic potential energy:

$$2F \cdot x_2 = \frac{1}{2}k(2x_2)^2$$
$$2Fx_2 = 2kx_2^2$$
$$F = kx_2$$
$$x_2 = F/k$$

**Method 2 - Force analysis:** Each mass experiences force F. The total spring extension is $2x_2$ ($x_2$ on each side).

$$F = k(\text{total extension})/2 = k(2x_2)/2 = kx_2$$
$$x_2 = F/k$$

**Actually, let's reconsider:** The spring extends by $x_2$ on each side, so total extension = $2x_2$ For the entire spring: $F = k(2x_2)$ But this force is applied at one end relative to the other.

**Correct analysis:** In case (b), due to symmetry, the center of the spring remains stationary. Each half acts like an independent spring with spring constant 2k (since each half has half the length).

For each mass: $F = (2k)x_2$

Therefore: $x_2 = F/(2k)$

**Maximum extension in case (b):** $x_2 = F/(2k)$

## (b) If the masses are released, what is the period of oscillation in each case?

**Case (a): One end fixed**

$$T_1 = 2\pi\sqrt{(m/k)}$$

**Case (b): Both ends free Method:** Each mass oscillates as if connected to a spring of constant 2k (effective spring constant for each half).

$$T_2 = 2\pi\sqrt{(m/2k)} = 2\pi\sqrt{(m/2k)} = (1/\sqrt{2}) \times 2\pi\sqrt{(m/k)}$$
$$T_2 = T_1/\sqrt{2}$$

**Alternative approach - Reduced mass:** In case (b), we can use the concept of reduced mass for two identical masses connected by a spring:

$\mu = m_1m_2/(m_1 + m_2) = m \cdot m/(m + m) = m/2$
$T_2 = 2\pi\sqrt{(\mu/k)} = 2\pi\sqrt{((m/2)/k)} = (1/\sqrt{2}) \times 2\pi\sqrt{(m/k)}$

### Summary:

- **(a) Maximum extensions:**
  - Case (a): $x_1 = F/k$
  - Case (b): $x_2 = F/(2k)$
- **(b) Periods:**
  - Case (a): $T_1 = 2\pi\sqrt{(m/k)}$
  - Case (b): $T_2 = T_1/\sqrt{2} = 2\pi\sqrt{(m/2k)}$

**Physical interpretation:**

- Case (b) has smaller extension because both ends contribute to spring force
- Case (b) has shorter period because effective spring constant is higher

---

## 13.14 The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rad/min, what is its maximum speed?

### Given Data:

- Stroke = 2A = 1.0 m
- Angular frequency ω = 200 rad/min
- SHM of the piston

## Solution:

### Finding amplitude:

$$\text{Stroke} = 2A = 1.0 \text{ m}$$
$$A = 0.5 \text{ m}$$

### Converting angular frequency to rad/s:

$$\omega = 200 \text{ rad/min} = 200/60 \text{ rad/s} = 10/3 \text{ rad/s}$$

### Maximum speed in SHM:

$$v_{max} = \omega A = (10/3) \times 0.5 = 5/3 \text{ m/s} \approx 1.67 \text{ m/s}$$

## Verification:

### Using energy method:

$$\text{Total energy } E = \tfrac{1}{2}kA^2$$
$$\text{At mean position: } E = \tfrac{1}{2}mv_{max}^2$$
$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{max}^2$$
$$v_{max} = A\sqrt{(k/m)} = A\omega \ \checkmark$$

## Physical Context:

- **Stroke** is the total distance traveled by piston (twice amplitude)
- **Maximum speed** occurs at the middle of the stroke
- **Zero speed** occurs at the ends of the stroke

**Answer: The maximum speed is 1.67 m/s**

---

## 13.15 The acceleration due to gravity on the surface of moon is 1.7 m $s^{-2}$. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? (g on the surface of earth is 9.8 m $s^{-2}$)

### Given Data:

- $g_{earth} = 9.8\ m/s^2$
- $g_{moon} = 1.7\ m/s^2$
- $T_{earth} = 3.5$ s
- Same pendulum (same length L)

### Solution:

**Simple pendulum formula:**

```
T = 2π√(L/g)
```

**For the same pendulum on Earth and Moon:**

```
T_earth = 2π√(L/g_earth)
T_moon = 2π√(L/g_moon)
```

**Taking ratio:**

```
T_moon/T_earth = √(g_earth/g_moon)
```

**Calculating:**

```
T_moon = T_earth × √(g_earth/g_moon)
T_moon = 3.5 × √(9.8/1.7)
T_moon = 3.5 × √5.76
T_moon = 3.5 × 2.4
T_moon = 8.4 s
```

### Verification:

```
Ratio = T_moon/T_earth = 8.4/3.5 = 2.4
√(g_earth/g_moon) = √(9.8/1.7) = √5.76 = 2.4 ✓
```

**Physical Interpretation:**

- Lower gravity on moon means weaker restoring force
- Weaker restoring force leads to slower oscillations
- Period increases by factor of $\sqrt{g_{earth}/g_{moon}}$

**Answer: The time period on moon is 8.4 s**

---

## 13.16 A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

**Given Data:**

- Pendulum length: l

- Bob mass: M
- Car moves in circle of radius R with speed v
- Oscillations in radial direction
- Small oscillations

## Analysis:

**Forces in the car's reference frame:**

1. **Gravitational force:** Mg (downward)
2. **Centrifugal force:** $Mv^2/R$ (radially outward)

**Equilibrium position:** The pendulum will hang at an angle $\theta_0$ from vertical where:

$$\tan \theta_0 = (Mv^2/R)/(Mg) = v^2/(Rg)$$

**Effective gravity:** The effective gravitational field in the car's frame is:

$$g\_eff = \sqrt{(g^2 + (v^2/R)^2)}$$

**For small oscillations about equilibrium:** The restoring force component is proportional to displacement, with effective gravity g_eff.

**Time period formula:**

$$T = 2\pi\sqrt{(l/g\_eff)} = 2\pi\sqrt{(l/\sqrt{(g^2 + v^4/R^2)})}$$

**Simplifying:**

$$T = 2\pi\sqrt{l/\sqrt{g^2 + v^4/R^2}}$$

## Alternative Approach:

### Components of effective acceleration:

- Vertical: g
- Radial (horizontal): $v^2/R$

### Magnitude of effective acceleration:

$$g_{eff} = \sqrt{g^2 + (v^2/R)^2}$$

### Time period:

$$T = 2\pi\sqrt{l/g_{eff}} = 2\pi\sqrt{l/\sqrt{g^2 + v^4/R^2}}$$

## Special Cases:

### Case 1: v = 0 (stationary car)

$$T = 2\pi\sqrt{l/g} \quad \text{[Normal pendulum formula]}$$

### Case 2: $v^2/R >> g$ (very fast circular motion)

$$T \approx 2\pi\sqrt{lR/v^2}$$

### Case 3: $v^2/R << g$ (slow circular motion)

$$T \approx 2\pi\sqrt{(l/g)}[1 + v^4/(2R^2g^2)]$$

**Answer:**

$$T = 2\pi\sqrt{(l/\sqrt{(g^2 + v^4/R^2)})}$$

---

## 13.17 A cylindrical piece of cork of density ρ, base area A and height h floats in a liquid of density $\rho_l$. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period $T = 2\pi\sqrt{(h\rho/g\rho_l)}$ where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).

### Given Data:

- Cork: density ρ, base area A, height h
- Liquid: density $\rho_l$
- Cork floats in equilibrium
- Small vertical displacement and release

### Analysis:

**Equilibrium Condition:** For floating cork, weight = buoyant force

$$\text{Weight} = \rho A h g$$
$$\text{Buoyant force} = \rho_l A h_0 g \quad (h_0 = \text{submerged height})$$
$$\rho A h g = \rho_l A h_0 g$$
$$h_0 = \rho h/\rho_l$$

**Displaced Position:** Let cork be pushed down by small distance x from equilibrium.

**New submerged volume:** $A(h_0 + x)$ **New buoyant force:** $\rho_l g A(h_0 + x)$

**Net force on cork:**

$$F_{net} = \text{Buoyant force} - \text{Weight}$$
$$F_{net} = \rho_l g A(h_0 + x) - \rho A h g$$
$$F_{net} = \rho_l g A h_0 + \rho_l g A x - \rho A h g$$

**At equilibrium:** $\rho_l g A h_0 = \rho A h g$, so:

$$F_{net} = \rho_l g A x$$

**But this force is upward (restoring), so:**

$$F_{net} = -\rho_l g A x \quad \text{(negative because it opposes displacement)}$$

**Equation of motion:**

$$Ma = -\rho_l g A x$$
$$\rho A h(d^2x/dt^2) = -\rho_l g A x$$
$$d^2x/dt^2 = -(\rho_l g/\rho h)x$$

**Comparing with SHM equation $d^2x/dt^2 = -\omega^2 x$:**

$$\omega^2 = \rho_l g/(\rho h)$$
$$\omega = \sqrt{\rho_l g/(\rho h)}$$

**Time period:**

$$T = 2\pi/\omega = 2\pi\sqrt{\rho h/(\rho_l g)}$$

**Rearranging:**

$$T = 2\pi\sqrt{h\rho/(g\rho_l)}$$

## Verification:

**Dimensional analysis:**

$$[T] = \sqrt{[h][\rho]/[g][\rho_l]} = \sqrt{L \cdot ML^{-3}/LT^{-2} \cdot ML^{-3}} = \sqrt{L/LT^{-2}} = \sqrt{T^2} = T \checkmark$$

**Physical check:**

- Higher cork density $\rho$ → longer period (more inertia)
- Higher liquid density $\rho_l$ → shorter period (stronger restoring force)
- Greater height h → longer period (more inertia)

## Answer: Proved that $T = 2\pi\sqrt{h\rho/g\rho_l}$

---

# 13.18 One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.

## Given Setup:

- U-tube with mercury

- One end connected to suction pump (pressure $P_1$)
- Other end open to atmosphere (pressure $P_0$)
- Initial pressure difference: $\Delta P = P_0 - P_1$
- Mercury column displaced from equilibrium

### Analysis:

**Initial Equilibrium with Pump:** Let the mercury level difference be $2h_0$ initially ($h_0$ higher on atmospheric side).

```
Pressure difference = ρgh(2h₀)
P₀ - P₁ = ρg(2h₀)
```

**When pump is removed:** Both ends are at atmospheric pressure $P_0$. Mercury will oscillate about new equilibrium (equal levels).

**Let's define coordinates:**

- Total mercury column length = $L$
- Displacement from equilibrium = $x$
- Left column rises by $x$, right column falls by $x$
- Height difference = $2x$

**Forces on mercury column:**

1. **Pressure force:** Negligible (both ends at $P_0$)
2. **Gravitational restoring force:** Due to height difference

**Restoring force analysis:** When displaced by $x$:

- Left column has excess mercury of height $x$

- Right column has deficit of mercury height x
- Net gravitational force = weight of excess mercury column

**Force calculation:**

```
Excess volume on left = Ax
Mass of excess mercury = ρAx
Weight = ρAxg
This weight acts downward (restoring force)
```

**But we need to consider the complete system:** The mercury column acts as a single entity of total mass M = ρAL.

**Pressure due to height difference:**

```
ΔP = ρg(2x)
Force = ΔP × A = ρgA(2x)
```

**This force acts to restore equilibrium:**

```
F_restoring = -ρgA(2x) = -2ρgAx
```

**Equation of motion:**

```
Ma = -2ρgAx
ρAL(d²x/dt²) = -2ρgAx
d²x/dt² = -(2g/L)x
```

**Comparing with SHM equation:**

$$\omega^2 = 2g/L$$
$$\omega = \sqrt{(2g/L)}$$

**Period of oscillation:**

$$T = 2\pi/\omega = 2\pi\sqrt{(L/2g)}$$

## Alternative Derivation:

**Energy method:** When mercury is displaced by x, the potential energy change is:

$$\Delta PE = mg(\text{height change of center of mass})$$

**For mercury column:**

- Left side: mass $\rho Ax$ rises by average height $x/2$
- Right side: mass $\rho Ax$ falls by average height $x/2$
- Net PE change = $\rho Ax \cdot g \cdot x = \rho gAx^2$

**Total energy:** $E = \frac{1}{2}mv^2 + \rho gAx^2$ For SHM: $E = \frac{1}{2}m\omega^2A^2$, comparing gives same $\omega$.

## Physical Interpretation:

- Mercury oscillates due to gravitational restoring force
- Period depends on total column length L
- Independent of tube cross-section A
- Similar to simple pendulum but with different geometry

**Answer:**

The mercury column executes SHM with:

$$\omega = \sqrt{2g/L}$$
$$T = 2\pi\sqrt{L/2g}$$

where L is the total length of mercury column in the U-tube.

---

## COMPLETE SOLUTIONS SUMMARY

This comprehensive answer key covers all 18 exercises from Chapter 13: Oscillations with detailed mathematical derivations, physical interpretations, and verification steps. Each solution includes:

1. **Clear problem identification**
2. **Step-by-step mathematical solution**
3. **Physical reasoning and interpretation**
4. **Dimensional analysis where appropriate**
5. **Verification of results**
6. **Alternative solution methods where relevant**

The solutions progress from basic concepts (periodic motion identification) to advanced applications (U-tube mercury oscillations), providing complete understanding for competitive exam preparation.