Chapter 14: Waves - NCERT Exercise Solutions

Physics Class 11

14.1 String Wave Speed

Given: String mass = 2.50 kg, tension = 200 N, length = 20.0 m

Solution:

Linear mass density: $\mu = m/L = 2.50/20.0 = 0.125$ kg/m
Wave speed: $v = \sqrt{(T/\mu)} = \sqrt{(200/0.125)} = \sqrt{1600} = 40$ m/s
Time = distance/speed = 20.0/40 = 0.5 s

Answer: Time = 0.5 s

14.2 Stone Drop and Sound Travel

Given: Tower height = 300 m, sound speed = 340 m/s, g = 9.8 m/s^2

Solution:

Time for stone to fall: $t_1 = \sqrt{(2h/g)} = \sqrt{(2\times300/9.8)} = 7.82$ s
Time for sound to travel up: $t_2 = h/v = 300/340 = 0.88$ s
Total time = $t_1 + t_2 = 7.82 + 0.88 = 8.7$ s

Answer: Splash heard after 8.7 s

14.3 Steel Wire Tension for Sound Speed

Given: Wire length = 12.0 m, mass = 2.10 kg, desired speed = 343 m/s

Solution:

Linear mass density: $\mu = m/L = 2.10/12.0 = 0.175$ kg/m
From $v = \sqrt{(T/\mu)}$: $T = \mu v^2 = 0.175 \times (343)^2 = 20{,}600$ N

Answer: Required tension = 20.6 kN

14.4 Sound Speed in Air - Conceptual

Using $v = \sqrt{(\gamma P/\rho)}$:

(a) Independence of pressure: At constant temperature, $P \propto \rho$ (ideal gas law), so P/ρ remains constant

(b) Increases with temperature: At constant pressure, ρ decreases with T, so $v \propto \sqrt{T}$

(c) Increases with humidity: Water vapor is less dense than dry air, reducing overall density and increasing speed

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14.5 Traveling Wave Functions

For $y = f(x \pm vt)$ to represent traveling wave:

(a) $(x - vt)^2$: YES - Function of $(x - vt)$, represents wave traveling in +x direction

(b) $\log[(x + vt)/x_0]$: YES - Function of (x + vt), represents wave traveling in -x direction

(c) 1/(x + vt): YES - Function of (x + vt), represents wave traveling in -x direction

All three can represent traveling waves

14.6 Bat Ultrasonic Wave Reflection/Transmission

Given: f = 1000 kHz, v_air = 340 m/s, v_water = 1486 m/s

Solution:

Wavelength in air: λ_air = v_air/f = $340/(1000\times10^3) = 3.4\times10^{-4}$ m

(a) Reflected sound: Same medium, so λ = 3.4×10^{-4} m = 0.34 mm

(b) Transmitted sound: λ_water = v_water/f = $1486/(1000\times10^3) = 1.486\times10^{-3}$ m = 1.49 mm

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14.7 Medical Ultrasound Scanner

Given: f = 4.2 MHz, v_tissue = 1.7 km/s = 1700 m/s

Solution:

Wavelength: λ = v/f = $1700/(4.2\times10^6) = 4.05\times10^{-4}$ m = 0.405 mm

Answer: λ = 0.41 mm in tissue

14.8 Transverse Wave Analysis

Given: $y(x,t) = 3.0 \sin(36t + 0.018x + \pi/4)$ cm

Solution: Comparing with $y = a \sin(\omega t + kx + \varphi)$:

(a) Traveling wave: YES, moving in -x direction (ω and k have same sign)

Speed: $v = \omega/k = 36/0.018 = 2000$ cm/s = 20 m/s (leftward)

(b) Amplitude = 3.0 cm, Frequency = $\omega/2\pi = 36/2\pi = 5.73$ Hz

(c) Initial phase at origin = $\pi/4$

(d) Wavelength: $\lambda = 2\pi/k = 2\pi/0.018 = 349$ cm

14.9 Wave Motion Comparison

For wave at different positions ($x = 0, 2, 4$ cm):

Shapes: All sinusoidal with same frequency and amplitude

Differences: Only **phase** differs between points - amplitude and frequency are identical for all positions in a traveling wave.

14.10 Phase Difference Calculation

Given: $y(x,t) = 2.0 \cos 2\pi(10t - 0.0080x + 0.35)$

Solution: Phase = $2\pi(10t - 0.0080x + 0.35)$ Phase difference = $2\pi \times 0.0080 \times \Delta x$

(a) $\Delta x = 4$ m: $\Delta\varphi = 2\pi \times 0.0080 \times 4 = 0.201$ rad

(b) $\Delta x = 0.5$ m: $\Delta\varphi = 2\pi \times 0.0080 \times 0.5 = 0.0251$ rad

(c) $\lambda = 2\pi/k = 125$ m, so $\lambda/2 = 62.5$ m: $\Delta\varphi = \pi$ rad

(d) $3\lambda/4 = 93.75$ m: $\Delta\varphi = 3\pi/2$ rad

14.11 Standing Wave Analysis

Given: $y(x,t) = 0.06 \sin(2\pi x/3) \cos(120\pi t)$

Solution: (a) Standing wave - variables x and t appear separately

(b) Superposition interpretation:

Wavelength: $\lambda = 2\pi/k = 2\pi/(2\pi/3) = 3$ m
Frequency: $f = \omega/2\pi = 120\pi/2\pi = 60$ Hz
Speed: $v = f\lambda = 60 \times 3 = 180$ m/s

(c) String tension:

$\mu = m/L = 0.03/1.5 = 0.02$ kg/m
$T = \mu v^2 = 0.02 \times (180)^2 = 648$ N

14.12 Standing Wave Properties

(a) Frequency: YES - All points oscillate with same frequency

(b) Phase: NO - Points between nodes have same phase, but different from points in adjacent segments

(c) Amplitude: NO - Amplitude varies as $|2a \sin(kx)|$ depending on position

Amplitude at x = 0.375 m:

$A = 0.06|\sin(2\pi \times 0.375/3)| = 0.06|\sin(\pi/4)| = 0.06 \times 0.707 = 0.042$ m

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14.13 Wave Type Classification

(a) $y = 2\cos(3x)\sin(10t)$: STATIONARY wave (x and t separate)

(b) $y = 2(x - vt)$: TRAVELING wave (function of x - vt)

(c) $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$: TRAVELING wave (both terms have same argument)

(d) $y = \cos x \sin t + \cos 2x \sin 2t$: NONE - Two different standing waves superposed

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14.14 String Fundamental Frequency

Given: f = 45 Hz, $m = 3.5\times10^{-2}$ kg, $\mu = 4.0\times10^{-2}$ kg/m

Solution:

Length: $L = m/\mu = 0.035/0.04 = 0.875$ m

(a) Wave speed: $v = f\lambda = f(2L) = 45 \times 2 \times 0.875 = 78.75$ m/s

(b) Tension: $T = \mu v^2 = 0.04 \times (78.75)^2 = 248$ N

14.15 Air Column Resonance

Given: f = 340 Hz, resonance at L = 25.5 cm and 79.3 cm

Solution:

For pipe closed at one end: L = (2n+1)λ/4
Difference: 79.3 - 25.5 = 53.8 cm = λ/2
Therefore: λ = 107.6 cm
Speed: v = fλ = 340 × 1.076 = 366 m/s

Answer: Sound speed = 366 m/s

14.16 Steel Rod Longitudinal Vibration

Given: L = 100 cm, clamped at middle, f = 2.53 kHz

Solution:

For rod clamped at center, L = λ/2
λ = 2L = 2.0 m
Speed: v = fλ = 2530 × 2.0 = 5060 m/s

Answer: Sound speed in steel = 5.06 km/s

14.17 Pipe Resonance Analysis

Given: L = 20 cm, f = 430 Hz, v = 340 m/s

Solution:

Wavelength: $\lambda = v/f = 340/430 = 0.791$ m = 79.1 cm

Closed pipe: $L = (2n+1)\lambda/4$
$20 = (2n+1) \times 79.1/4$
$n = 0.01$ (not integer - no resonance)

Open pipe: $L = n\lambda/2$
$20 = n \times 79.1/2$
$n = 0.51$ (not integer - no resonance)

Answer: No resonance in either case

14.18 Beat Frequency Analysis

Given: Beat frequency = 6 Hz, tension in A reduced, beats become 3 Hz, $f_A = 324$ Hz

Solution:

Initially: $|f_A - f_B| = 6$ Hz
So $f_B = 324 \pm 6 = 330$ Hz or 318 Hz

Reducing tension decreases frequency of A.
If $f_B = 330$ Hz: beat frequency would increase
If $f_B = 318$ Hz: beat frequency would decrease to 3 Hz ✓

Therefore: $f_B = 318$ Hz

14.19 Conceptual Explanations

(a) Displacement node = pressure antinode: In sound waves, where particles don't move (displacement node), pressure variation is maximum (pressure antinode) and vice versa.

(b) Bat echolocation: Bats emit ultrasound and analyze reflected waves to determine distance, direction, size and nature of objects - biological sonar.

(c) Same frequency, different sounds: Instruments have different harmonic content (overtones) giving characteristic timbres despite same fundamental frequency.

(d) Wave types in different media: Solids can support both shear (transverse) and compression (longitudinal) waves. Fluids cannot sustain shear stress, only longitudinal waves possible.

(e) Pulse distortion: In dispersive media, different frequencies travel at different speeds, causing pulse shape to change during propagation.

Key Formulas Reference

Wave Properties

- **Wave speed:** $v = f\lambda = \omega/k$
- **String waves:** $v = \sqrt{(T/\mu)}$
- **Sound waves:** $v = \sqrt{(\gamma P/\rho)} = \sqrt{(B/\rho)}$

Wave Equations

- **General:** $y(x,t) = a \sin(kx - \omega t + \varphi)$
- **Standing wave:** $y(x,t) = 2a \sin(kx) \cos(\omega t)$

Resonance

- **String (both ends fixed):** $f_n = nv/(2L)$
- **Pipe (one end closed):** $f_n = (2n+1)v/(4L)$
- **Pipe (both ends open):** $f_n = nv/(2L)$

Beats

- **Beat frequency:** $f_{beat} = |f_1 - f_2|$