Chapter 1: Units and Measurement

Comprehensive Study Notes

Class 11 Physics - NCERT Based

EXAM SPRINT - Complete Coverage for JEE and NEET Examinations

1.1 INTRODUCTION

What is Measurement?

Definition: Measurement of any physical quantity involves comparison with a certain basic, arbitrarily chosen, internationally accepted reference standard called **unit**.

Expression of Measurement:

> Result = Number (Numerical measure) + Unit

Example: Length = 5.2 meters

- **5.2** = Numerical measure
- **meters** = Unit

Types of Physical Quantities

1. Fundamental/Base Quantities

Definition: Basic physical quantities that cannot be expressed in terms of other quantities
Examples: Length, Mass, Time, Electric Current, Temperature, Amount of Substance, Luminous Intensity

2. Derived Quantities

Definition: Physical quantities that can be expressed as combinations of fundamental quantities
Examples: Area, Volume, Speed, Force, Energy, Density

Units Classification

1. Fundamental/Base Units

Definition: Units for fundamental quantities **Characteristics:**

- Independent of other units
- Arbitrarily chosen
- Internationally accepted standards

2. Derived Units

Definition: Units obtained by combining base units **Formation:** Mathematical combinations of base units **Examples:**

- Area = length × length → m^2
- Speed = length/time → m/s
- Force = mass × acceleration → $kg \cdot m/s^2$

System of Units

Definition: Complete set of base units and derived units used for measurement

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1.2 THE INTERNATIONAL SYSTEM OF UNITS (SI)

Historical Systems

1. CGS System

Base Units:

- **Length:** Centimeter (cm)
- **Mass:** Gram (g)
- **Time:** Second (s)

2. FPS (British) System

Base Units:

- **Length:** Foot (ft)
- **Mass:** Pound (lb)
- **Time:** Second (s)

3. MKS System

Base Units:

- **Length:** Meter (m)
- **Mass:** Kilogram (kg)
- **Time:** Second (s)

SI System (Système Internationale d'Unites)

Development

Organization: Bureau International des Poids et Mesures (BIPM) **Established:** 1971 **Recent Revision:** November 2018

Advantages of SI

1. **Decimal System:** Easy conversions

2. **International Acceptance:** Worldwide usage
3. **Scientific Consistency:** Based on fundamental constants
4. **Commercial Convenience:** Universal standards

Seven SI Base Units

Quantity	Unit Name	Symbol	Key Definition Concept
Length	meter	m	Speed of light in vacuum
Mass	kilogram	kg	Planck constant
Time	second	s	Caesium-133 atom frequency
Electric Current	ampere	A	Elementary charge
Temperature	kelvin	K	Boltzmann constant
Amount of Substance	mole	mol	Avogadro constant
Luminous Intensity	candela	cd	Luminous efficacy

Detailed SI Base Unit Definitions

1. Meter (m) - Length

Definition: Defined by fixing the speed of light in vacuum **Value:** c = 299,792,458 m/s (exact)
Previous Standard: Platinum-iridium bar (obsolete)

2. Kilogram (kg) - Mass

Definition: Defined by fixing the Planck constant **Value:** $h = 6.62607015 \times 10^{-34}$ J·s (exact)
Previous Standard: International Prototype Kilogram (obsolete)

3. Second (s) - Time

Definition: Based on caesium-133 atom transitions **Value:** $\Delta\nu cs$ = 9,192,631,770 Hz (exact)
Physical Basis: Hyperfine transition frequency

4. Ampere (A) - Electric Current

Definition: Defined by fixing the elementary charge **Value:** $e = 1.602176634 \times 10^{-19}$ C (exact) **Relationship:** C = A·s

5. Kelvin (K) - Temperature

Definition: Defined by fixing the Boltzmann constant **Value:** $k = 1.380649 \times 10^{-23}$ J/K (exact) **Reference Point:** Triple point of water

6. Mole (mol) - Amount of Substance

Definition: Contains exactly $6.02214076 \times 10^{23}$ elementary entities **Value:** NA = $6.02214076 \times 10^{23}$ mol^{-1} (Avogadro constant) **Note:** Elementary entities must be specified (atoms, molecules, ions, etc.)

7. Candela (cd) - Luminous Intensity

Definition: Defined by fixing luminous efficacy of monochromatic radiation **Frequency:** 540×10^{12} Hz **Value:** Kcd = 683 lm/W (exact)

Supplementary SI Units

1. Plane Angle - Radian (rad)

Definition: θ = arc length/radius = s/r **Nature:** Dimensionless **Full Circle:** 2π radians = 360°

2. Solid Angle - Steradian (sr)

Definition: Ω = intercepted area/radius^2 = A/r^2 **Nature:** Dimensionless **Full Sphere:** 4π steradians

SI Prefixes for Multiples and Sub-multiples

Prefix	Symbol	Factor	Example
tera	T	10^{12}	1 THz = 10^{12} Hz
giga	G	10^{9}	1 GB = 10^{9} bytes
mega	M	10^{6}	1 MHz = 10^{6} Hz
kilo	k	10^{3}	1 km = 10^{3} m
centi	c	10^{-2}	1 cm = 10^{-2} m
milli	m	10^{-3}	1 mm = 10^{-3} m
micro	μ	10^{-6}	1 μm = 10^{-6} m
nano	n	10^{-9}	1 nm = 10^{-9} m
pico	p	10^{-12}	1 pm = 10^{-12} m

Some Non-SI Units Still in Use

Quantity	Unit	Symbol	SI Equivalent
Time	minute	min	60 s
Time	hour	h	3600 s
Time	day	d	86,400 s
Angle	degree	°	(π/180) rad
Volume	liter	L	10^{-3} m^3
Mass	metric ton	t	10^{3} kg

1.3 SIGNIFICANT FIGURES

Introduction to Measurement Errors

Key Concept: Every measurement involves errors due to:

1. **Instrument limitations** (least count)
2. **Environmental conditions**
3. **Human error**
4. **Random fluctuations**

Definition of Significant Figures

Significant Figures: All digits in a measurement that are known reliably plus the first digit that is uncertain.

Example:

- **1.62 s:** Three significant figures (1, 6 certain; 2 uncertain)
- **287.5 cm:** Four significant figures (2, 8, 7 certain; 5 uncertain)

Rules for Determining Significant Figures

Rule 1: Non-zero Digits

All non-zero digits are significant

- 1234 → 4 significant figures
- 56.78 → 4 significant figures

Rule 2: Zeros Between Non-zero Digits

All zeros between non-zero digits are significant

- 1002 → 4 significant figures
- 50.06 → 4 significant figures

Rule 3: Leading Zeros

Zeros to the left of the first non-zero digit are NOT significant

- 0.0023 → 2 significant figures (2, 3)
- 0.00456 → 3 significant figures (4, 5, 6)

Rule 4: Trailing Zeros Without Decimal

Trailing zeros in numbers without decimal point are NOT significant

- 1200 → 2 significant figures (1, 2)
- 45000 → 2 significant figures (4, 5)

Rule 5: Trailing Zeros With Decimal

Trailing zeros in numbers with decimal point are significant

- 12.00 → 4 significant figures
- 0.2500 → 4 significant figures

Scientific Notation and Significant Figures

Advantages of Scientific Notation

1. **Eliminates ambiguity** about trailing zeros
2. **Clearly shows** significant figures
3. **Simplifies** very large or small numbers
4. **Standardizes** representation

Format: $a \times 10^b$

- **a:** Number between 1 and 10
- **b:** Integer exponent
- **All digits in 'a' are significant**

Examples:

- 4700 m = 4.7×10^3 m (2 sig figs)
- 4700. m = 4.700×10^3 m (4 sig figs)
- 0.00340 = 3.40×10^{-3} (3 sig figs)

Order of Magnitude

Definition: Power of 10 when number is expressed approximately as 10^b

Determination:

- If coefficient (a) ≤ 5: round down
- If coefficient (a) > 5: round up to next power of 10

Examples:

- Earth's diameter: 1.28×10^7 m → Order of magnitude: 10^7
- Hydrogen atom: 1.06×10^{-10} m → Order of magnitude: 10^{-10}

1.3.1 Rules for Arithmetic Operations

Rule 1: Multiplication and Division

Result should have the same number of significant figures as the measurement with the fewest significant figures

Example:

- 4.237 g ÷ 2.51 cm^3 = 1.69 g/cm^3 (3 sig figs)
- 3.00×10^8 m/s × 3.1557×10^7 s = 9.47×10^{15} m (3 sig figs)

Rule 2: Addition and Subtraction

Result should have the same number of decimal places as the measurement with the fewest decimal places

Examples:

- 436.32 g + 227.2 g + 0.301 g = 663.8 g (1 decimal place)
- 0.307 m - 0.304 m = 0.003 m (3 decimal places)

1.3.2 Rounding Off Rules

Standard Rounding Rules:

1. **If digit > 5:** Round up
    - 2.746 → 2.75 (3 sig figs)
2. **If digit < 5:** Round down
    - 1.743 → 1.74 (3 sig figs)
3. **If digit = 5:** Round to even
    - 2.745 → 2.74 (preceding digit 4 is even)
    - 2.735 → 2.74 (preceding digit 3 is odd, round up)

Multi-step Calculations:

- **Keep one extra digit** in intermediate steps
- **Round to proper significant figures** only at the end

- **Prevents accumulation** of rounding errors

1.3.3 Uncertainty in Arithmetic Operations

Addition/Subtraction Error Combination

Absolute errors add

- If l = 16.2 ± 0.1 cm and b = 10.1 ± 0.1 cm
- Area = lb = 163.62 ± 2.6 cm^2
- Final result: 164 ± 3 cm^2

Multiplication/Division Error Combination

Relative errors add

- Relative error in l = 0.6%
- Relative error in b = 1.0%
- Relative error in lb = 1.6%

Significant Figure Dependency on Magnitude

Same absolute error, different relative errors:

- 1.02 g ± 0.01 g → Relative error = ±1%
- 9.89 g ± 0.01 g → Relative error = ±0.1%

1.4 DIMENSIONS OF PHYSICAL QUANTITIES

Definition of Dimensions

Dimensions: The powers (exponents) to which the base quantities are raised to represent a physical quantity.

Notation: Square brackets [ ] indicate "dimensions of"

- [Length] = [L]
- [Mass] = [M]
- [Time] = [T]

Seven Fundamental Dimensions

Physical Quantity	Dimension	SI Unit
Length	[L]	m
Mass	[M]	kg
Time	[T]	s
Electric Current	[A]	A
Temperature	[K]	K
Amount of Substance	[mol]	mol
Luminous Intensity	[cd]	cd

Mechanical Quantities (In terms of [M], [L], [T])

Examples:

1. **Volume:** $[L] \times [L] \times [L] = [L^3]$
2. **Speed:** $[L]/[T] = [LT^{-1}]$
3. **Acceleration:** $[L]/[T^2] = [LT^{-2}]$
4. **Force:** $[M][L]/[T^2] = [MLT^{-2}]$
5. **Density:** $[M]/[L^3] = [ML^{-3}T^0]$

Important Dimensional Formulas

Physical Quantity	Formula	Dimensions
Area	length × breadth	$[L^2]$
Volume	length × breadth × height	$[L^3]$
Velocity	displacement/time	$[LT^{-1}]$
Acceleration	velocity/time	$[LT^{-2}]$
Force	mass × acceleration	$[MLT^{-2}]$
Pressure	force/area	$[ML^{-1}T^{-2}]$
Work/Energy	force × displacement	$[ML^2T^{-2}]$
Power	work/time	$[ML^2T^{-3}]$
Momentum	mass × velocity	$[MLT^{-1}]$
Impulse	force × time	$[MLT^{-1}]$

1.5 DIMENSIONAL FORMULAE AND DIMENSIONAL EQUATIONS

Dimensional Formula

Definition: Expression showing how and which base quantities represent the dimensions of a physical quantity.

Examples:

- Volume: $[M^0L^3T^0]$
- Speed: $[M^0LT^{-1}]$
- Force: $[MLT^{-2}]$
- Density: $[ML^{-3}T^0]$

Dimensional Equation

Definition: Equation obtained by equating a physical quantity with its dimensional formula.

Format: [Physical Quantity] = [Dimensional Formula]

Examples:

- $[V] = [M^0L^3T^0]$
- $[v] = [M^0LT^{-1}]$
- $[F] = [MLT^{-2}]$
- $[\rho] = [ML^{-3}T^0]$

Deriving Dimensional Formulas

From Definition:

Density = Mass/Volume

- $[\rho] = [M]/[L^3] = [ML^{-3}T^0]$

From Equations:

Newton's Second Law: F = ma

- $[F] = [M][LT^{-2}] = [MLT^{-2}]$

1.6 DIMENSIONAL ANALYSIS AND ITS APPLICATIONS

Principle of Homogeneity

Statement: Physical quantities can only be added or subtracted if they have the same dimensions.

Mathematical Equations: All terms must have the same dimensions.

1.6.1 Checking Dimensional Consistency

Process:

1. **Write dimensions** of each term
2. **Check if all terms** have same dimensions
3. **Verify LHS = RHS** dimensionally

Example: Kinematic Equation

Equation: $x = x_0 + v_0 t + ½at^2$

Dimensional Check:

- [x] = [L]

$[x_0] = [L]$
$[v_0 t] = [LT^{-1}][T] = [L]$
$[½at^2] = [LT^{-2}][T^2] = [L]$

Result: All terms have dimension [L] → Equation is dimensionally consistent

Limitations of Dimensional Analysis:

1. **Cannot determine dimensionless constants**
2. **Cannot verify exact numerical relationships**
3. **Cannot distinguish between quantities with same dimensions**
4. **Trigonometric, logarithmic, exponential functions must have dimensionless arguments**

1.6.2 Deducing Relations Among Physical Quantities

Method:

1. **Identify variables** on which quantity depends
2. **Assume product-type dependence**
3. **Apply principle of dimensional homogeneity**
4. **Solve for exponents**

Example: Simple Pendulum Time Period

Given: T depends on length (l), mass (m), and gravity (g)

Assume: $T = k\, l^x g^y m^z$ (k = dimensionless constant)

Dimensional Analysis:

- $[T] = [L^x][LT^{-2}]^y[M^z]$
- $[T] = [L^{(x+y)}][T^{(-2y)}][M^z]$

Equating Powers:

- For [M]: $z = 0$
- For [T]: $-2y = 1 \rightarrow y = -1/2$
- For [L]: $x + y = 0 \rightarrow x = 1/2$

Result: $T = k\sqrt{(l/g)}$ where $k = 2\pi$

Applications of Dimensional Analysis:

1. **Check equation correctness**
2. **Derive relationships**
3. **Convert units**
4. **Estimate orders of magnitude**

Limitations:

1. **Cannot determine pure numbers**
2. **Limited to product-type dependencies**
3. **Cannot handle trigonometric relationships**
4. **Maximum 3-4 independent variables**

JEE/NEET Specific Important Points

High-Yield Topics:

1. SI Base Units:

- **Names, symbols, definitions**
- **Recent redefinitions (2018)**
- **Fundamental vs derived quantities**

2. Significant Figures:

- **Rules for counting**
- **Arithmetic operations rules**
- **Scientific notation**
- **Order of magnitude**

3. Dimensional Analysis:

- **Dimensional formulas**
- **Checking equation consistency**
- **Deriving relationships**

- **Unit conversions**

4. Error Analysis:

- **Absolute vs relative errors**
- **Error propagation in calculations**
- **Precision vs accuracy**

Common JEE/NEET Question Types:

1. Significant Figures Problems:

- Counting significant figures
- Arithmetic with significant figures
- Scientific notation conversions

2. Dimensional Analysis:

- Finding dimensional formula
- Checking equation validity
- Deriving unknown relationships

3. Unit Conversions:

- Between different systems
- Using dimensional analysis
- Prefix conversions

4. Error Calculations:

- Percentage errors

- Combination of errors
- Relative error problems

Memory Aids and Mnemonics

SI Base Units:

"My Little Tiny Ant Truly Ate Many Cookies"

- **M**eter (Length)
- **L**... → Wait, let me fix this:

"Many Little Tigers Always Take Much Lunch"

- **M**eter (Length)
- **L**... Let me try again:

"Length Mass Time Amp Temp Amount Light"

- **L**ength → Meter
- **M**ass → Kilogram
- **T**ime → Second
- **A**mpere → Current
- **T**emperature → Kelvin
- **A**mount → Mole
- **L**uminous → Candela

Significant Figure Rules:

"All Non-zero, Between Significant, Leading Not, Trailing Maybe"

- **A**ll non-zero digits significant
- **B**etween non-zero digits significant
- **L**eading zeros not significant
- **T**railing zeros maybe (depends on decimal)

Common Dimensional Formulas:

"Force MLT-2, Energy ML2T-2, Power ML2T-3"

Order of Magnitude Examples:

- **Human height:** 10^0 m (1 m)
- **Room size:** 10^1 m (10 m)
- **Building height:** 10^2 m (100 m)
- **City size:** 10^4 m (10 km)
- **Earth radius:** 10^7 m
- **Atom size:** 10^{-10} m

Practice Problems for JEE/NEET

Significant Figures:

1. **How many significant figures:** 0.02370 g? **Answer:** 4 (2, 3, 7, 0)
2. **Calculate with proper significant figures:** 4.237 × 2.51 ÷ 1.6 **Answer:** 6.6 (2 significant figures)
3. **Express in scientific notation:** 0.000456 with proper significant figures **Answer:** 4.56×10^{-4} (3 significant figures)

Dimensional Analysis:

1. **Check dimensional consistency:** $v^2 = u^2 + 2as$ **Solution:** $[LT^{-1}]^2 = [LT^{-1}]^2 + [LT^{-2}][L] = [L^2T^{-2}]$ ✓
2. **Find dimensions of:** Gravitational constant G in $F = Gm_1m_2/r^2$ **Answer:** $[G] = [M^{-1}L^3T^{-2}]$
3. **Derive formula:** Time period of simple pendulum depends on l, g, m **Answer:** $T \propto \sqrt{(l/g)}$, independent of mass

Unit Conversion:

1. **Convert:** 72 km/h to m/s **Answer:** 72 × (1000/3600) = 20 m/s
2. **Express:** 1 J in CGS units **Answer:** 1 J = 10^7 erg

Error Analysis:

1. **If l = 10.0 ± 0.1 cm, find relative error Answer:** (0.1/10.0) × 100% = 1%

Summary Tables

SI Base Quantities Quick Reference:

Quantity	Symbol	Unit	Symbol
Length	l	meter	m
Mass	m	kilogram	kg
Time	t	second	s
Current	I	ampere	A
Temperature	T	kelvin	K
Amount	n	mole	mol
Luminous Intensity	I	candela	cd

Common Derived Units:

Quantity	Unit	Symbol	In Base Units
Force	newton	N	$kg \cdot m \cdot s^{-2}$
Energy	joule	J	$kg \cdot m^2 \cdot s^{-2}$
Power	watt	W	$kg \cdot m^2 \cdot s^{-3}$
Pressure	pascal	Pa	$kg \cdot m^{-1} \cdot s^{-2}$
Frequency	hertz	Hz	s^{-1}

Significant Figure Summary:

Type of Zero	Significant?	Example
Leading	No	0.0023 (2 sig figs)
Between non-zero	Yes	1002 (4 sig figs)
Trailing (no decimal)	No	1200 (2 sig figs)
Trailing (with decimal)	Yes	12.00 (4 sig figs)

Advanced Topics for JEE

Dimensional Analysis Limitations:

1. **Cannot determine dimensionless constants** (like 2π, ½)
2. **Cannot verify trigonometric relationships**
3. **Limited to power law relationships**
4. **Cannot distinguish between similar quantities** (work vs torque)

Error Analysis Types:

1. **Systematic Errors:** Consistent, predictable
2. **Random Errors:** Unpredictable fluctuations
3. **Gross Errors:** Human mistakes
4. **Instrumental Errors:** Due to instrument limitations

Precision vs Accuracy:

- **Precision:** Reproducibility of measurements
- **Accuracy:** Closeness to true value
- **Can have high precision but low accuracy**

EXAM SPRINT - Master Units and Measurement with focused study on SI units, significant figures, dimensional analysis, and error calculations. Regular practice of numerical problems and unit conversions is essential for JEE/NEET success.

Source: NCERT Physics Class 11, Chapter 1 - Comprehensive coverage for competitive exam preparation