Chapter 14: Waves - Detailed Notes

14.1 INTRODUCTION

What Are Waves?

Definition: Patterns of disturbance that move without actual physical transfer of matter as a whole

Key Characteristics:

- **Energy Transport**: Waves carry energy from one point to another
- **Information Transfer**: Pattern of disturbance contains information
- **No Mass Transfer**: Medium doesn't flow with the wave
- **Communication Basis**: All communication depends on wave transmission

Everyday Examples:

- **Water Waves**: Cork pieces move up-down, not outward with circles
- **Sound Waves**: Air doesn't flow from speaker to ear
- **Seismic Waves**: Ground vibrations during earthquakes
- **Light Waves**: Energy reaches us from distant stars

Types of Waves:

1. Mechanical Waves

- **Require medium**: Cannot propagate through vacuum
- **Examples**: Sound waves, water waves, seismic waves, waves on strings

- **Mechanism**: Oscillations of medium particles via elastic forces
- **Depend on**: Elastic properties and inertia of medium

2. Electromagnetic Waves

- **Don't require medium**: Can travel through vacuum
- **Examples**: Light, radio waves, X-rays, microwaves
- **Speed in vacuum**: c = 299,792,458 m/s (constant for all EM waves)
- **Study**: Covered in Class XII

3. Matter Waves

- **Associated with**: Particles like electrons, protons, atoms
- **Quantum mechanical**: Abstract but technologically important
- **Applications**: Electron microscopes
- **Study**: Advanced quantum mechanics

Physical Mechanism of Wave Propagation:

Spring Chain Model:

- Connected springs demonstrate wave propagation
- Disturbance travels without springs moving bodily
- Each spring oscillates about equilibrium position
- **Analogy**: Train bogies with spring coupling

Sound Wave Mechanism:

- **Compression/Rarefaction**: Creates density changes ($\delta\rho$)
- **Pressure Changes**: δp proportional to density changes

- **Restoring Force**: Pressure differences drive oscillations
- **Propagation**: Disturbance moves region to region

Solid Wave Mechanism:

- **Lattice Structure**: Atoms arranged periodically
- **Equilibrium Forces**: Each atom balanced by neighbors
- **Elastic Restoring Force**: Displacement creates spring-like forces
- **Model**: Atoms as masses connected by springs

14.2 TRANSVERSE AND LONGITUDINAL WAVES

Transverse Waves

Definition: Particle oscillation perpendicular to wave propagation direction

Characteristics:

- **Particle Motion**: ⊥ to wave direction
- **Medium Requirement**: Only in media that can sustain shear stress
- **Possible in**: Solids (not in fluids under normal conditions)
- **Example**: Waves on stretched string

String Wave Analysis:

- Elements oscillate up-down (y-direction)
- Wave travels along string (x-direction)
- **Fixed time**: Shows wave shape in space
- **Fixed position**: Shows particle oscillation in time

Longitudinal Waves

Definition: Particle oscillation parallel to wave propagation direction

Characteristics:

- **Particle Motion**: ∥ to wave direction
- **Medium Requirement**: Any medium with bulk elasticity
- **Possible in**: Solids, liquids, and gases
- **Example**: Sound waves in air

Sound Wave Mechanism:

- **Piston Motion**: Creates compressions and rarefactions
- **Density Variations**: Higher/lower than equilibrium
- **Pressure Waves**: Propagate through medium
- **No Air Flow**: Only pressure disturbance moves

Medium Requirements:

Wave Type	Stress Type	Media Possible
Transverse	Shear	Solids only
Longitudinal	Compressive	Solids, liquids, gases

Special Cases:

Water Surface Waves:

- **Capillary Waves**: Short wavelength (few cm), surface tension restoring force
- **Gravity Waves**: Long wavelength (meters to hundreds of meters), gravity restoring force

- **Complex Motion**: Combined longitudinal and transverse components
- **Not purely transverse**: Despite appearance

Wave Speeds:

- Generally different speeds for transverse and longitudinal waves in same medium
- Depends on different elastic properties

14.3 DISPLACEMENT RELATION IN A PROGRESSIVE WAVE

Mathematical Description

Sinusoidal Traveling Wave:

$$y(x,t) = a \sin(kx - \omega t + \varphi)$$

Alternative Form:

$$y(x,t) = A \sin(kx - \omega t) + B \cos(kx - \omega t)$$

Where: $a = \sqrt{(A^2 + B^2)}$ and $\tan \varphi = B/A$

Physical Interpretation:

Wave Traveling in +x Direction:

$$y(x,t) = a \sin(kx - \omega t + \varphi)$$

Wave Traveling in -x Direction:

```
y(x,t) = a sin(kx + ωt + φ)
```

Parameter Definitions:

Amplitude (a):

- **Physical Meaning**: Maximum displacement from equilibrium
- **Always Positive**: By convention
- **Range**: y varies between +a and -a
- **Units**: Same as displacement (meters)

Phase:

- **Definition**: $(kx - \omega t + \varphi)$
- **Determines**: Displacement at any position and time
- **Initial Phase**: φ (at $x = 0$, $t = 0$)
- **Phase Difference**: Determines interference effects

14.3.1 Amplitude and Phase

Amplitude Properties:

- Represents maximum particle displacement
- Independent of position and time for progressive wave
- Determined by energy of wave source
- Same for all particles in ideal progressive wave

Phase Significance:

- Determines instantaneous displacement

- Controls interference between waves
- Can be set to zero by choice of origin
- Phase differences create interference patterns

14.3.2 Wavelength and Angular Wave Number

Wavelength (λ):

Definition: Minimum distance between two points having same phase

Mathematical Derivation:

For same displacement at positions x and $x + \lambda$:

$$\sin(kx - \omega t + \varphi) = \sin(k(x + \lambda) - \omega t + \varphi)$$

This requires: $k\lambda = 2\pi$

Therefore: $\lambda = 2\pi/k$

Angular Wave Number (k):

$$k = 2\pi/\lambda$$

- **Units**: rad/m or m^{-1}
- **Physical Meaning**: 2π times number of waves per unit length
- **Also Called**: Propagation constant

14.3.3 Period, Angular Frequency and Frequency

Period (T):

Definition: Time for one complete oscillation

For particle at $x = 0$:

$$y(0,t) = a \sin(-\omega t) = -a \sin(\omega t)$$

Same displacement after time T:

$$-a \sin(\omega t) = -a \sin(\omega(t + T))$$

This requires: $\omega T = 2\pi$

Therefore: $T = 2\pi/\omega$

Angular Frequency (ω):

$$\omega = 2\pi/T$$

- **Units**: rad/s
- **Physical Meaning**: Rate of phase change

Frequency (ν):

$$\nu = 1/T = \omega/(2\pi)$$

- **Units**: Hz (hertz)
- **Physical Meaning**: Number of oscillations per second

Longitudinal Wave Description:

For longitudinal waves, displacement function:

$$s(x,t) = a \sin(kx - \omega t + \varphi)$$

Where $s(x,t)$ = displacement parallel to propagation direction

---

14.4 THE SPEED OF A TRAVELLING WAVE

General Speed Formula

Wave Speed: Rate at which wave pattern moves

Derivation from Phase Condition:

For constant phase: $kx - \omega t = \text{constant}$

Differentiating: $k(dx/dt) - \omega = 0$

Therefore: $\mathbf{v = \omega/k}$

Alternative Forms:

$$v = \omega/k = \lambda\nu = \lambda/T$$

Universal Relation:

$$v = \lambda\nu$$

Physical Meaning: In time T (one period), wave travels distance λ (one wavelength)

Wave Speed Dependencies:

- **Medium Properties**: Elastic properties and inertia

- **Not Wave Properties**: Independent of λ, ν, or amplitude
- **Source Determines**: Frequency of wave generated
- **Medium Determines**: Wave speed
- **Speed and Frequency**: Together determine wavelength

14.4.1 Speed of Transverse Wave on Stretched String

Dimensional Analysis Approach:

Relevant Quantities:

- Tension: T $[MLT^{-2}]$
- Linear mass density: $\mu = m/L$ $[ML^{-1}]$

Dimensional Check:

$$[T/\mu] = [MLT^{-2}]/[ML^{-1}] = [L^2T^{-2}]$$

Therefore: $\sqrt{T/\mu}$ has dimensions $[LT^{-1}]$

Result:

$$v = \sqrt{T/\mu}$$

Physical Interpretation:

- **Higher Tension**: Faster wave (stronger restoring force)
- **Higher Mass Density**: Slower wave (more inertia)
- **String Properties**: Completely determine speed

Wavelength Determination:

$$\lambda = v/\nu = \sqrt{(T/\mu)}/\nu$$

14.4.2 Speed of Longitudinal Waves (Sound)

Bulk Modulus Approach:

Relevant Quantities:

- Bulk Modulus: $B = -\Delta P/(\Delta V/V)$ $[ML^{-1}T^{-2}]$
- Mass Density: ρ $[ML^{-3}]$

Dimensional Check:

$$[B/\rho] = [ML^{-1}T^{-2}]/[ML^{-3}] = [L^2T^{-2}]$$

General Formula:

$$v = \sqrt{(B/\rho)}$$

For Solid Bars:

$$v = \sqrt{(Y/\rho)}$$

Where Y = Young's modulus

Speed Comparison:

Medium Type	Typical Speed (m/s)	Reason
Gases	300-500	Low bulk modulus
Liquids	1000-1500	Higher bulk modulus
Solids	3000-6000	Highest bulk modulus

Sound in Ideal Gas:

Newton's Formula (Isothermal):

$$v = \sqrt{P/\rho}$$

- **Problem**: 15% error compared to experiment
- **Assumption**: Isothermal pressure changes

Laplace Correction (Adiabatic): For adiabatic process: $PV^\gamma = \text{constant}$

Adiabatic bulk modulus: $B_{ad} = \gamma P$

Therefore: $\mathbf{v = \sqrt{\gamma P/\rho}}$

- **γ**: Ratio of specific heats (C_p/C_v)
- **For air**: $\gamma = 7/5 = 1.4$
- **Result**: Agrees with experimental value (331 m/s at STP)

14.5 THE PRINCIPLE OF SUPERPOSITION OF WAVES

Statement:

When two or more waves pass through same medium simultaneously, net displacement at any point is algebraic sum of individual displacements

Mathematical Expression:

$$y(x,t) = y_1(x,t) + y_2(x,t) + ... + y_n(x,t)$$

Or: $y = \Sigma_i f_i(x - vt)$

Key Principles:

- **Wave Independence**: Each wave moves as if others aren't present
- **Linear Addition**: Displacements add algebraically
- **Temporary Effect**: During overlap only
- **Wave Identity**: Waves retain identity after crossing

Interference of Harmonic Waves

Setup:

Two harmonic waves with same ω, k, amplitude a:

$$y_1(x,t) = a \sin(kx - \omega t)$$
$$y_2(x,t) = a \sin(kx - \omega t + \varphi)$$

Resultant Wave:

$$y(x,t) = y_1 + y_2 = a[\sin(kx - \omega t) + \sin(kx - \omega t + \varphi)]$$

Using trigonometric identity:

$$y(x,t) = 2a \cos(\varphi/2) \sin(kx - \omega t + \varphi/2)$$

Resultant Amplitude:

$$A(\varphi) = 2a \cos(\varphi/2)$$

Interference Cases:

Constructive Interference (φ = 0, 2π, 4π, ...):

$$A = 2a \text{ (maximum)}$$
$$y(x,t) = 2a \sin(kx - \omega t)$$

- Waves in phase
- Amplitudes add
- Maximum resultant amplitude

Destructive Interference (φ = π, 3π, 5π, ...):

$$A = 0 \text{ (minimum)}$$
$$y(x,t) = 0$$

- Waves completely out of phase
- Amplitudes cancel
- Zero resultant displacement

Partial Interference (other φ values):

$$0 \le A \le 2a$$

- Intermediate amplitudes

- Depends on phase difference

14.6 REFLECTION OF WAVES

Boundary Conditions

Rigid Boundary:

- **Condition**: Zero displacement at boundary
- **Phase Change**: π (180°)
- **Amplitude**: Same as incident (no energy loss)

Incident Wave: $y_i(x,t) = a \sin(kx - \omega t)$ **Reflected Wave**: $y_r(x,t) = -a \sin(kx + \omega t)$

Free/Open Boundary:

- **Condition**: Maximum displacement allowed
- **Phase Change**: 0
- **Amplitude**: Same as incident

Reflected Wave: $y_r(x,t) = a \sin(kx + \omega t)$

Physical Explanation:

Rigid Boundary:

- Boundary exerts reaction force
- Newton's third law creates phase reversal
- Total displacement at boundary = 0

Free Boundary:

- No constraint on boundary motion
- No reaction force
- No phase reversal

14.6.1 Standing Waves and Normal Modes

Formation:

Superposition of incident and reflected waves of same amplitude and frequency

Mathematical Description:

Incident: $y_1(x,t) = a \sin(kx - \omega t)$ **Reflected**: $y_2(x,t) = a \sin(kx + \omega t)$

Resultant:

$$y(x,t) = y_1 + y_2 = 2a \sin(kx) \cos(\omega t)$$

Characteristics of Standing Waves:

Key Differences from Traveling Waves:

- **No wave propagation**: Pattern doesn't move
- **Amplitude varies spatially**: $2a \sin(kx)$
- **All particles oscillate in phase**: Same ω
- **Fixed nodes and antinodes**: Positions don't change

Nodes and Antinodes:

Nodes (Zero amplitude):

- Condition: $\sin(kx) = 0$

- Positions: $x = n\pi/k = n\lambda/2$ where n = 0,1,2,3,...
- **Spacing**: λ/2 between consecutive nodes

Antinodes (Maximum amplitude):

- Condition: $\sin(kx) = \pm 1$
- Positions: $x = (n + 1/2)\pi/k = (n + 1/2)\lambda/2$ where n = 0,1,2,3,...
- **Spacing**: λ/2 between consecutive antinodes

Normal Modes of Strings

String Fixed at Both Ends:

Boundary Conditions: x = 0 and x = L are nodes

Allowed Wavelengths:

$$L = n(\lambda/2) \rightarrow \lambda = 2L/n$$

where n = 1,2,3,... (positive integers)

Normal Mode Frequencies:

$$\nu_n = nv/(2L) = n\sqrt{(T/\mu)}/(2L)$$

Harmonics:

- **n = 1**: Fundamental frequency (first harmonic)
- **n = 2**: Second harmonic
- **n = 3**: Third harmonic, etc.

Normal Modes of Air Columns

Pipe Closed at One End:

Boundary Conditions:

- $x = 0$: Node (closed end)
- $x = L$: Antinode (open end)

Allowed Wavelengths:

$$L = (n + 1/2)(\lambda/2) \rightarrow \lambda = 4L/(2n + 1)$$

where $n = 0,1,2,3,...$

Normal Mode Frequencies:

$$\nu_n = (2n + 1)v/(4L)$$

Only Odd Harmonics: $\nu_1, 3\nu_1, 5\nu_1, 7\nu_1,...$

Pipe Open at Both Ends:

Boundary Conditions: Both ends are antinodes

Normal Mode Frequencies:

$$\nu_n = nv/(2L)$$

All Harmonics Present: $\nu_1, 2\nu_1, 3\nu_1, 4\nu_1,...$

Resonance:

When external frequency matches normal mode frequency, system exhibits resonance with large amplitude oscillations.

14.7 BEATS

Definition:

Beats: Periodic variation in amplitude when two waves of slightly different frequencies interfere

Mathematical Analysis

Two Sound Waves:

$$s_1 = a\cos(\omega_1 t)$$
$$s_2 = a\cos(\omega_2 t)$$

Resultant:

$$s = s_1 + s_2 = a[\cos(\omega_1 t) + \cos(\omega_2 t)]$$

Using trigonometric identity:

$$s = 2a\cos(\omega_\beta t)\cos(\omega_a t)$$

Where:

- $\omega_\beta = (\omega_1 - \omega_2)/2$ (beat frequency)
- $\omega_a = (\omega_1 + \omega_2)/2$ (average frequency)

Physical Interpretation:

$$s = [2a \cos(\omega_\beta t)] \cos(\omega_a t)$$

- **Amplitude Modulation**: $2a \cos(\omega_\beta t)$
- **Carrier Frequency**: ω_a
- **Amplitude varies** with frequency $2\omega_\beta = \omega_1 - \omega_2$

Beat Frequency:

$$\nu_{\beta eat} = |\nu_1 - \nu_2|$$

Applications:

Musical Tuning:

- Musicians use beats to tune instruments
- Eliminate beats by matching frequencies
- Sensitive method for frequency comparison

Frequency Measurement:

- Unknown frequency determined by beat frequency
- Reference frequency provides calibration

Beat Pattern Characteristics:

- **Constructive interference**: Maximum amplitude when waves in phase
- **Destructive interference**: Minimum amplitude when waves out of phase
- **Beat period**: Time between successive maxima = $1/|\nu_1 - \nu_2|$

SUMMARY OF KEY FORMULAS

Wave Equation:

$$y(x,t) = a \sin(kx - \omega t + \varphi)$$

Wave Parameters:

- **Wavelength**: $\lambda = 2\pi/k$
- **Period**: $T = 2\pi/\omega$
- **Frequency**: $\nu = \omega/(2\pi) = 1/T$
- **Wave Speed**: $v = \omega/k = \lambda\nu$

Wave Speeds:

- **String**: $v = \sqrt{(T/\mu)}$
- **Sound in fluid**: $v = \sqrt{(B/\rho)}$
- **Sound in solid**: $v = \sqrt{(Y/\rho)}$
- **Sound in gas**: $v = \sqrt{(\gamma P/\rho)}$

Standing Waves:

$$y(x,t) = 2a \sin(kx) \cos(\omega t)$$

Normal Modes:

- **String (both ends fixed)**: $\nu_n = nv/(2L)$
- **Pipe (one end closed)**: $\nu_n = (2n+1)v/(4L)$

- **Pipe (both ends open)**: $\nu_n = nv/(2L)$

Beats:

$$\nu_{\beta eat} = |\nu_1 - \nu_2|$$