Chapter 5: Work, Energy and Power

5.1 SCALAR PRODUCT (DOT PRODUCT)

Definition: $\vec{A} \cdot \vec{B} = AB \cos \theta$

Properties:

- Result is a scalar quantity
- Commutative: $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
- Distributive: $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$
- $\vec{A} \cdot (\lambda\vec{B}) = \lambda(\vec{A} \cdot \vec{B})$

Component Form: $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$

Unit Vector Properties:

- $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$
- $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

Geometric Interpretation:

- $\vec{A} \cdot \vec{B} = A(B \cos \theta) = B(A \cos \theta)$
- Product of one vector's magnitude and other's component along it

5.2 WORK

Definition: Work done by force $\vec{F}$ over displacement $\vec{d}$: $W = \vec{F} \cdot \vec{d} = Fd \cos \theta$

Key Points:

- Scalar quantity

- Units: Joule (J) = N·m
- Dimensions: $[ML^2T^{-2}]$

Sign of Work:

- Positive: $0° \leq \theta < 90°$ (force assists motion)
- Negative: $90° < \theta \leq 180°$ (force opposes motion)
- Zero: $\theta = 90°$ (force perpendicular to displacement)

No Work Done When:

1. Displacement is zero (d = 0)
2. Force is zero (F = 0)
3. Force perpendicular to displacement ($\theta = 90°$)

Work by Variable Force: W = ∫[xi to xf] F(x)dx

5.3 KINETIC ENERGY

Definition: $K = ½mv^2$

Properties:

- Scalar quantity
- Always positive
- Units: Joule (J)
- Measure of work an object can do by virtue of its motion

5.4 WORK-ENERGY THEOREM

Statement: Change in kinetic energy = Work done by net force

Mathematical Form: $K_f - K_i = W_{net}$ $\Delta K = W_{net}$

For Variable Force: $K_f - K_i = \int_{x_i}^{x_f} F(x)dx$

Key Insights:

- Integral form of Newton's Second Law
- Scalar form (directional information lost)
- Useful for solving problems without knowing detailed force variation

5.5 POTENTIAL ENERGY

Definition: Energy stored by virtue of position or configuration

Mathematical Relation: $F(x) = -dV(x)/dx$

Conservative Force: Force derivable from potential energy function

- Work path-independent
- Work zero over closed path
- Examples: Gravity, spring force

Gravitational PE: $V(h) = mgh$ (Near earth's surface, $h << R_E$)

Elastic PE (Spring): $V(x) = \frac{1}{2}kx^2$ (Hooke's Law: $F_s = -kx$)

5.6 CONSERVATION OF MECHANICAL ENERGY

Statement: For conservative forces only: Total mechanical energy = Kinetic + Potential = Constant

Mathematical Form: $E = K + V = \text{constant}$ $K_i + V_i = K_f + V_f$

Conditions:

- Only conservative forces do work
- No friction or other dissipative forces

For Non-Conservative Forces: $E_f - E_i = W_{nc}$ (Where W_{nc} = work by non-conservative forces)

5.7 POWER

Definition: Rate of doing work or transferring energy

Average Power: $P_{av} = W/t$

Instantaneous Power: $P = dW/dt = \vec{F} \cdot \vec{v}$

Units:

- SI: Watt (W) = J/s
- Practical: Horse Power (hp)
- 1 hp = 746 W

Energy Units:

- kWh = 3.6×10^6 J
- Used in electricity bills

5.8 COLLISIONS

Key Principles:

1. **Momentum Conservation:** Always conserved in collisions
2. **Energy Considerations:** KE may or may not be conserved

Types of Collisions:

Elastic Collision:

- Kinetic energy conserved
- Momentum conserved
- Objects separate after collision

For 1D Elastic Collision (m2 initially at rest): $v_{1f} = (m_1 - m_2)/(m_1 + m_2) \times v_{1i}$ $v_{2f} = 2m_1/(m_1 + m_2) \times v_{1i}$

Special Cases:

- Equal masses: $v_{1f} = 0$, $v_{2f} = v_{1i}$
- $m_2 >> m_1$: $v_{1f} \approx -v_{1i}$, $v_{2f} \approx 0$

Inelastic Collision:

- Kinetic energy not conserved
- Momentum conserved
- Some KE converted to other forms

Completely Inelastic:

- Objects stick together after collision
- Maximum KE loss for given collision

For 1D Completely Inelastic: $v_f = m_1v_{1i}/(m_1 + m_2)$

Energy Loss: $\Delta K = (m_1m_2)/(2(m_1 + m_2)) \times v_{1i}^2$

2D Collisions:

- Momentum conserved in both directions
- For elastic: Energy also conserved

- Generally need additional information to solve

IMPORTANT FORMULAS SUMMARY

Work and Energy:

- $W = \vec{F} \cdot \vec{d} = Fd \cos \theta$
- $K = ½mv^2$
- $\Delta K = Wnet$

Potential Energy:

- $V(h) = mgh$ (gravitational)
- $V(x) = ½kx^2$ (elastic)
- $F = -dV/dx$

Power:

- $P = W/t$ (average)
- $P = \vec{F} \cdot \vec{v}$ (instantaneous)

Conservation:

- $E = K + V =$ constant (conservative forces)
- $Ef - Ei = Wnc$ (with non-conservative forces)

Collisions:

- Momentum: $\vec{p}i = \vec{p}f$ (always)
- Elastic: $Ki = Kf$ (kinetic energy conserved)
- Inelastic: $Ki > Kf$ (kinetic energy not conserved)

PROBLEM-SOLVING STRATEGIES

For Work Problems:

1. Identify all forces acting on object
2. Find displacement vector
3. Calculate work by each force using $W = \vec{F} \cdot \vec{d}$
4. Sum to get net work

For Energy Problems:

1. Identify conservative vs non-conservative forces
2. Choose appropriate energy conservation form
3. Set up energy equation at different points
4. Account for work by non-conservative forces

For Collision Problems:

1. Apply momentum conservation (vector equation)
2. For elastic collisions, also apply energy conservation
3. Use component form for 2D problems
4. Check if additional constraints given

Key Points to Remember:

- Work depends on force and displacement, not just force
- Energy is conserved only with conservative forces
- Power involves rate of energy transfer
- Momentum always conserved in collisions

- Choose reference points for potential energy wisely